DEFENCE
SCIENCE & TECHNOLOGY

A Strain-based Cohesive Zone Model for a Crack in a Power-Law Material under Grossly Plastic Conditions

G. X. Chen, C. H. Wang and
L. R. F. Rose

DSTO-RR-0235

DEFENCE
SCIENCE & TECHNOLOGY

A Strain-based Cohesive Zone Model for a Crack in a Power-Law Material under Grossly Plastic Conditions

G. X. Chen, C. H. Wang, and L. R. F. Rose

Airframes and Engines Division
Aeronautical and Maritime Research Laboratory

DSTO-RR-0235

ABSTRACT

To develop an analytical method for quantifying the growth behaviour of short cracks embedded in notch plastic zones for power law strain hardening materials, a strain-based cohesive zone model is proposed in which the conventional equilibrium equation in the stress-based model is replaced by strain compatibility. A Comparison with finite element results shows that this strain-based model provides accurate values of the crack-tip-opening displacement for applied strains up to four times the yield strain under general yielding conditions. Furthermore, it is shown that the cohesive stress determined by a method proposed in this work gives better results than the existing method, which are appropriate only for small-scale yielding conditions.

RELEASE LIMITATION

Approved for public release

AQ F02-12-3329

Published by

DSTO Aeronautical and Maritime Research Laboratory
506 Lorimer St
Fishermans Bend, Victoria 3207 Australia

Telephone: (03) 9626 7000
Fax: (03) 9626 7999

AR-012-297
April 2002

APPROVED FOR PUBLIC RELEASE

A Strain-based Cohesive Zone Model for a Crack in a Power-Law Material under Grossly Plastic Conditions

Executive Summary

Modelling of the growth behaviour of a short fatigue crack at a notch root, with the crack being engulfed by the notch plastic zone, remains a challenging issue, especially for power-law materials. Several schematic models have been put forward to explain and quantify the notch fatigue crack growth behaviour, including the effect of notch plasticity and the lack of plasticity-induced crack closure. Attempts have been made to explore the applicability of elastic-plastic fracture mechanics parameters, including the J-integral, strain-intensity factor, etc., showing only limited success.

For the case of elastic-perfectly plastic materials, a strain-based implementation of the Dugdale model has recently been developed for applications involving a small crack engulfed within a notch plastic zone, but this work is limited to the elastic-perfectly plastic material. In the present work, the strain-based Dugdale model is further extended to account for the effect of materials strain hardening. A comparison with finite element results shows that this strain-based model provides accurate values for the crack-tip-opening displacement for applied strains up to four times the yield strain under generally yielding conditions. Furthermore, a new method is proposed to determine the cohesive stress pertaining to strain hardening materials, which has been shown to provide better results than the existing methods, which are appropriate only for small-scale yielding condition.

Results of the present research provide a practical and physically plausible approach for extending the scope of current predictive software for fatigue crack growth based on the Dugdale model to power-law strain hardening materials, including conditions of large-scale yielding. This new method will enable a more accurate and reliable prediction of crack growth life, which provides the basis for setting the inspection intervals of aircraft.

Authors

Dr G. X. Chen
Airframes and Engines Division

G. X. Chen is currently a Research Scientist in the Airframes and Engines Division. Her experience has focused on the finite element and boundary element analysis, health monitoring, damage detection, manufacturing process, structural mechanics and dynamics. Since she joined DSTO in 1998, she has been working on the short crack modelling and shape optimisation. Before then, she had held various academic positions in universities of China and Japan, and CSIRO in Australia.

Dr C. H. Wang
Airframes and Engines Division

Dr. Chun H. Wang is currently a Principal Research Scientist and the Functional Head Damage Mechanics in the Airframes and Engines Division. He has made important contributions in fatigue and fracture mechanics, constitutive modelling, and advanced composite materials. Prior to joining DSTO in 1995, he has held various academic positions at the University of Sheffield, UK, the University of Sydney, and Deakin University, Australia.

Dr L. R. F. Rose
Airframes and Engines Division

Francis Rose graduated with a BSc (Hons.) from Sydney University in 1971 and a PhD from Sheffield University, UK, in 1975. He was appointed as a Research Scientist at the Aeronautical Research Laboratory in 1976, and is currently the Research Leader in Fracture Mechanics in Airframes and Engines Division. He has made important research contributions in fracture mechanics, non-destructive evaluation and applied mathematics. He is the regional editor for the international Journal of Fracture and a member of the editorial board of Mechanics of Materials. He is also a Fellow of the Institute for Applied Mathematics and its Application, UK, and a fellow of the Institution of Engineers, Australia.

Contents

NOMENCLATURE

a = crack length
b = any point between the original crack tip a and current crack tip c
c = length of fictitious crack $c=a+r_p$
d_n = displacement variable
E = Young's modulus
E' = generalised Young's modulus $=E/1-\upsilon^2$ for plain strain, E for plain stress
E_s = secant modulus
$G(x,c)$ = Green function
h = half height of the plate
k = strain gradient
K_σ , K_{σ_0} = stress-intensity factors
L = elastic-plastic boundary
r_p = plastic zone size
(r,θ)= polar coordinates centred at the crack tip
u_y = displacement in y direction
w = width of the plate
α = the plastic constraint factor
δ_M = crack tip opening displacement
$\varepsilon_{\max}$ = maximum applied strain
$\varepsilon_{ij}, \sigma_{ij}$ = strain and stress
$\varepsilon_{ep}, \sigma_{eq}$ = equivalent stress and strain
σ_Y = uniaxial yield stress
σ^*, σ_0^* = hypothetical stress distributions
σ_n = necking stress
σ_0 = cohesive stress
v = Poisson ratio
v_{ep} = the effective Poisson's ratio

1. Introduction

Modelling of the growth behaviour of a short fatigue crack at a notch root, with the crack being engulfed by the notch plastic zone, remains a challenging issue (Suresh, 1991; Shin, 1994). There are two major technical issues: (i) to develop a suitable correlating parameter for the crack growth rate, especially when the entire crack is embedded within grossly yielded material (Wang and Rose, 1999), and (ii) to develop computationally efficient and robust methods for computing this correlating parameter, taking into account notch plasticity. Several schematic models have been put forward to explain and quantify the notch fatigue crack growth behaviour, including a postulate the effect of notch plasticity (Smith and Miller, 1978) and the lack of plasticity-induced crack closure (Newman, 1982). Attempts have been made to explore the applicability of elastic-plastic fracture mechanics parameters, including the *J*-integral, strain-intensity factor, etc., showing only limited success (Shin, 1994). For stage II crack growth in a ductile material, the cyclic crack-tip opening displacement (ΔCTOD) has been shown to be a promising correlating parameter (Wang and Rose, 1999), capable of bridging the gap between mechanically small cracks (Wang and Rose, 1999) and long cracks (Guo *et al*, 1999). Here the usual operational definition for CTOD (Shih, 1981) is adopted. However, computation of the crack-tip opening displacement for a crack embedded in a notch plastic zone represents a significant challenge as detailed finite element calculations using extremely fine mesh and incremental plasticity theories are required. Therefore an analytical method is required to calculate the crack tip opening displacement.

Although the finite element method could be used for elastic-plastic stress analysis of growing fatigue cracks, thereby providing a basis for correlating and predicting crack growth rates for arbitrary geometry and loading conditions, the routine application of this approach is too computationally intensive and fraught with difficulties in detailed implementation to be a practical option. Instead, a more promising alternative is to use FE results to evaluate and to guide the interpretation or implementation of analytical models such as the Dugdale model and its variations (Dugdale, 1960; Rich and Roberts, 1968; Gowda, 1972; and Haddad, et al, 1978) and thereby to achieve a more computationally efficient predictive tool. Indeed, considerable success has been achieved already with this approach, most notably through the work of Newman (1982, 1992, 1998) and Galatolo (1996). However, it is recognised by Newman (1998) that his current implementation of the Dugdale model in FASTRAN II does not cater for cases of extensive notch plasticity. This limitation provided a major motivation for the present work.

In previous work (Wang, Chen and Rose 1999, 2002a), the stress-based Dugdale models were shown to predict values for the plastic zone size r_p and the crack tip opening displacement δ_M (CTOD) which agree closely with FE results for low to moderate strain levels. However, predictions diverged from the FE results at higher strain levels when gross yielding occurs. Furthermore, the strain intensity factor method was found to give slightly better estimates of CTOD than the stress-based Dugdale models, but significantly underestimate the plastic zone size.

A strain-based Dugdale model (Wang, Rose and Chen 2002b) was shown to give accurate values for both r_p and δ_M for strain levels up to four times yield strain,

with no evidence of significantly diminishing accuracy. This model can be applied equally for plane strain as for plane stress with a constraint factor. However the analysis was only limited to elastic-perfectly plastic material behaviour.

The aim of this report is to present a rational basis for extending this strain-based Dugdale model to the power-law material response under a specified remote strain. In order to simulate the crack behaviour at a notch root, the influence of a strain gradient become more important, in addition to strain-controlled remote loading. Accordingly, the simple case of an edge crack with different edge crack lengths subjected to a remote strain varying linearly along the crack path is considered here. The accuracy and range of applicability of this strain-based Dugdale model is assessed by direct comparison with finite-element (FE) computations detailed in Section 2. An analysis of the prospective stress in the uncracked plate is presented in Section 3, while Section 4 presents an extension of the Strain-Based Dugdale model.

2. Finite Element Analysis

There are two important features which need to be accounted for in modelling the plastic deformation associated with a small crack at a notch root: (i) a notch strain field which varies with distance from the notch root, and (ii) the effect of the free surface at the notch root on the in-plane constraint prevailing within the crack tip plastic zone.

To focus on these two features in their simplest form, consider an edge-cracked plate subjected to a linearly varying remote strain. The cracked plate has the following dimension: $h/w = 1$, and $a/w = 0.05$, as shown in Fig. 1, where the crack length is denoted as a, the width of the plate and the half of height of the plate as w and h, respectively. To simulate a linearly varying strain distribution, the top and bottom edges of the plate are displaced according to,

$$u_y(x) = h\varepsilon_{\max}(1+kx) \tag{1}$$

where $\varepsilon_{\max}$ denotes the maximum strain and k denotes the normalised strain gradient:

$$k = \frac{1}{\varepsilon_{\max}} d\varepsilon_{yy}(x)/dx \tag{2}$$

The remote applied strain is thus,

$$\varepsilon_{yy}(x) = \varepsilon_{\max}(1+kx) \tag{3}$$

In this work, the minimum strain at the right side is one fourth that of maximum strain and opposite in sign. Therefore, the strain gradient is $k = -5/4w$.

The computation was carried out using a finite-element code (ABAQUS, 1998). The material is assumed to obey a power-law strain hardening:

$$\frac{\varepsilon_{eq}}{\varepsilon_Y} = \frac{\sigma_{eq}}{\sigma_Y} + \alpha\left(\frac{\sigma_{eq}}{\sigma_Y}\right)^n \tag{4}$$

with Poisson ratio $\nu = 0.3$, and $\sigma_Y / E = 0.0005$. Here σ_Y denotes the uniaxial yield stress, $\sigma_{eq},\ \varepsilon_{eq}$ denote equivalent stress and strain, and α and n denote the material parameters. In the present work, various strain hardening components will be considered (n=3, 6, and 10) with a fixed hardening coefficient $\alpha = 0.01$. The deformation was assumed to be two-dimensional plane strain. Due to symmetry,

only half plate was analysed. Eight-noded quadrilateral elements were employed, and the mesh is shown in Fig. 2. The mesh near the crack tip was refined to capture the crack-tip plastic blunting behaviour.

The problem depicted in Fig. 1 has been analysed for power-law hardening materials (Al-Ani and Hancock, 1991), where a unique asymptotic field exists at the crack tip. It was found the crack-tip deformation is no longer dominated by the HRR field when plastic yielding engulfs the entire crack. This implies that the conventional elastic-plastic fracture mechanics approaches based on *J*-integral may fail to provide an adequate account of the near-tip plastic deformation. Therefore a new method is called for to provide an accurate assessment of the plastic deformation at the tip of small crack embedded in a fully plastic strain field.

For the shallow crack shown in Fig. 2, as the applied maximum strain increases, plastic deformation starts to develop in two separate regions, ahead of the crack tip and away from the crack, see Fig 3(a), (b). Finally, when a certain strain is reached, the two plastic regions merge into one, as shown in Fig. 3(c).

A measure of the crack-tip plastic deformation that has important implications for fatigue crack growth is the crack-tip opening displacement. The parameters of primary interest in the present work, for the purposes of comparison with the analytical estimates based on the Dugdale model, are the plastic zone size r_p and the crack-tip opening displacement δ_M. Numerical values for these parameters were derived from the FE results by the procedures shown below. The crack-tip opening displacement is defined as the crack face separation where the 45-degree lines drawn back from the crack-tip intersect the deformed crack face (Shih, 1981), as illustrated in Fig. 4, which can be mathematically expressed as

$$\delta_M = 2u_y(r^*, \theta = \pi) \tag{5}$$

where r^* is the root of the following equation,

$$r - u_x(r, \theta = \pi) = u_y(r, \theta = \pi) \tag{6}$$

where (r,θ) denote polar coordinates centred at the crack tip. As shown by Shih (1981), this construction gives a CTOD that matches the Dugdale model for the small-scale yielding case under plane stress conditions.

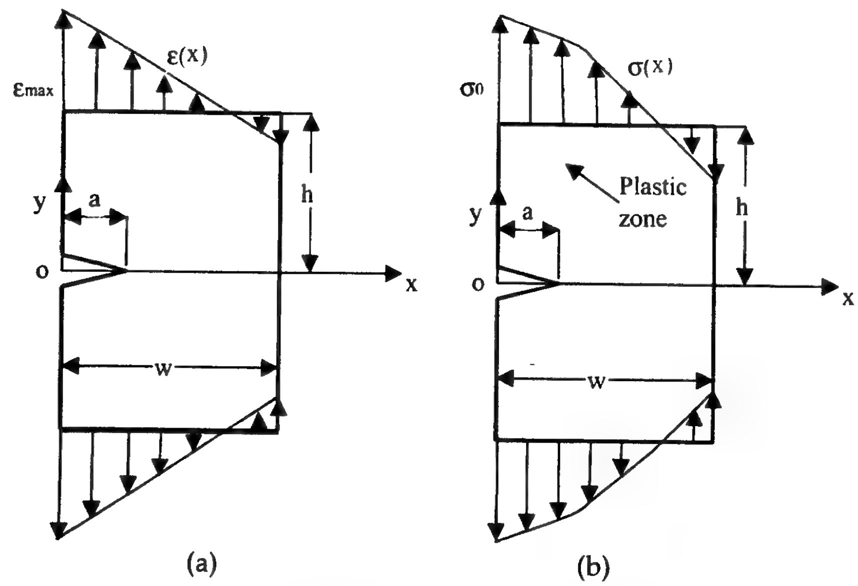


Figure 1 *An edge-cracked plate subjected to a linearly varying strain field showing (a) the strain distribution and (b) the attendant stress distribution for power-law strain hardening material.*

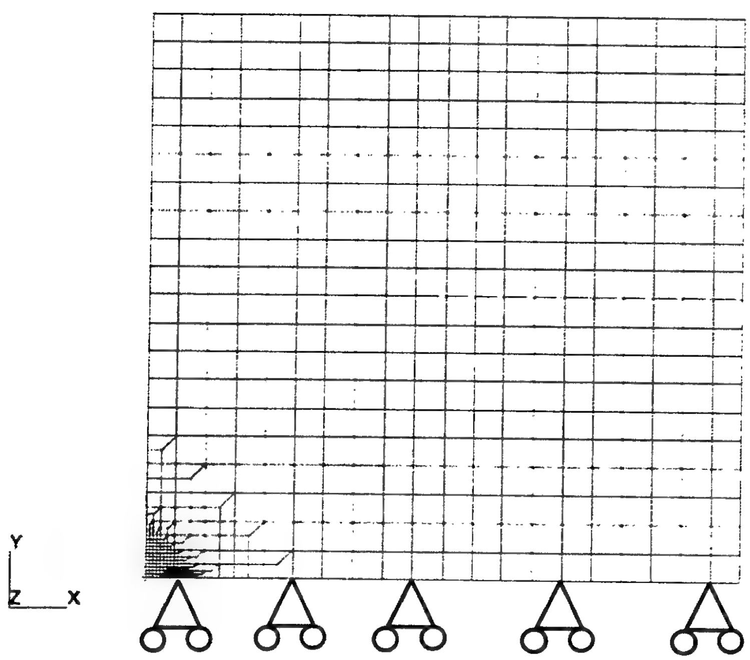
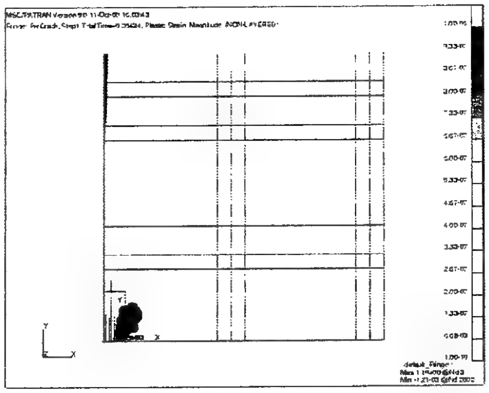
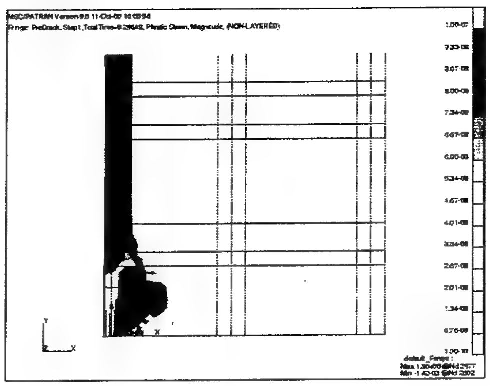


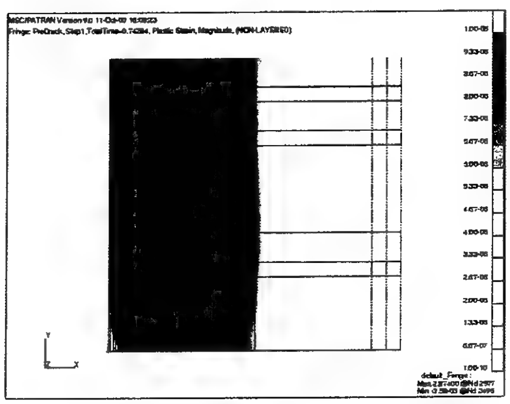
Figure 2 Finite element model of half plate



(a) $\varepsilon_{\max} / \varepsilon_0 = 1.06$



(b) $\varepsilon_{\max} / \varepsilon_0 = 1.17$



(c) $\varepsilon_{\max} / \varepsilon_0 = 2.96$

Figure 3 Plastic deformations under plane strain conditions: (a) front face yielding, (b) break-through, and (c) gross-section yielding

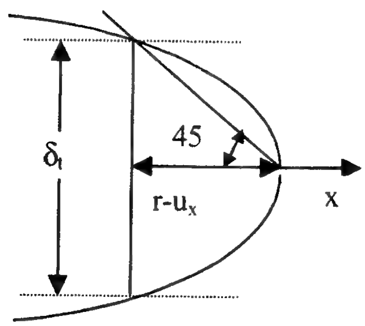


Figure 4 The definition of crack-tip opening displacement

3. Stress distribution in the un-cracked plate

For the nominal elastic stress distribution in an uncracked plate subjected to the linearly varying strain given by Eq (3), under the plane strain conditions, two issues need to be addressed: (i) accounting for the effects of stress triaxiality; (ii) selecting an appropriate constraint factor α for the limit stress σ_0 in the Dugdale model. The elastic plastic stress distribution in the absence of a crack can be calculated using Hencky's deformation theory of plasticity (Chakrabarty, 1987), this is because the deformation state in the uncracked plate is a proportional one: ratios of stress components and ratios of strain components are independent of applied strain.

For the plane strain case, $\varepsilon_{zz} = 0$, the non-zero strain components are: ε_{xx} and ε_{yy}, and the associated non-zero stress components are σ_{yy} and σ_{zz}. The condition of symmetry implies that $\sigma_{xy,y} = 0$ from the equilibrium equations $\sigma_{xx,x} = -\sigma_{xy,y} = 0$, so that in view of the symmetry condition $\sigma_{xy}(y=0) = 0$ and of the free-surface boundary condition $\sigma_{xx}(x=0) = 0$, one finds $\sigma_{xx} = \tau_{xy} = 0$ throughout the body. According to deformation theory of plasticity, the two non-zero stresses are related,

$$\sigma_{zz} = \nu_{ep}\sigma_{yy} \tag{7}$$

where ν_{ep} denotes the effective Poisson's ratio,

$$\nu_{ep} = 0.5 - (0.5 - \nu)\frac{E_s}{E} \tag{8}$$

Here E_s is the secant modulus, defined as:

$$E_s = \frac{\sigma_{eq}}{\varepsilon_{eq}} \tag{9}$$

The equivalent stress σ_{eq} is, by definition:

$$\sigma_{eq} = \sqrt{1-\nu_{ep}+\nu_{ep}^2}\,\sigma_{yy} \tag{10}$$

and according to the generalised Hooke's law:

$$\sigma_{yy} = \frac{E_s}{1-\nu_{ep}^{\ 2}}\varepsilon_{yy} \tag{11}$$

Substituting Eqs (9, 10) into Eq (11), we can get:

$$\varepsilon_{eq} = \frac{\sqrt{1-\nu_{ep}+\nu_{ep}^{\ 2}}}{1-\nu_{ep}^{\ 2}}\varepsilon_{yy} \tag{12}$$

Here, ν_{ep} needs to be determined iteratively. Inserting Eq (8-12) into Eq (4):

$$\alpha\left(\frac{\nu_{ep}-0.5}{0.5-\nu}\right)^{n}\left(\frac{\sqrt{1-\nu_{ep}+\nu_{ep}^2}}{1-\nu_{ep}^{\ 2}}\right)^{n-1}\left(\frac{\varepsilon_{yy}}{\varepsilon_Y}\right)^{n-1}+\frac{\nu_{ep}-0.5}{0.5-\nu}-1=0 \tag{13}$$

$\varepsilon_{yy}(x)$ is the applied far field strain from Eq (3) and $\nu = 0.3$, effective poison ratio ν_{ep} can be readily determined by solving the above non-linear equation. Then $\varepsilon_{eq}, \sigma_{yy}$ and σ_{eq} can be determined from Eq (12), Eq (11) and Eq (10) respectively. Therefore, the distribution of the elasto-plastic stress can thus be expressed as

$$\sigma_{eq}(x) = \begin{cases} \dfrac{\sigma_Y}{(1-\nu_{ep}+\nu_{ep}^2)^{1/2}} & , \quad x \le L \\ E'\varepsilon_{yy}(x) \quad , & \quad x > L \end{cases} \tag{14}$$

where E' denotes the generalised Young's modulus ($=E$ for generalised plane stress, or $E/(1-\nu^2)$ for plane strain, ν denoting Poisson's ratio), L represents the elastic-plastic boundary:

$$L = \frac{4w}{5}\left(1-\frac{\sigma_Y}{E'\varepsilon_{\max}}\right), \qquad \varepsilon_{\max} \ge \sigma_Y / E' \tag{15}$$

The comparison between the analytical and FE results is shown in Fig. 5. The elasto-plastic stress distribution, which would prevail in the absence of a crack, is independent of y for plane strain conditions.

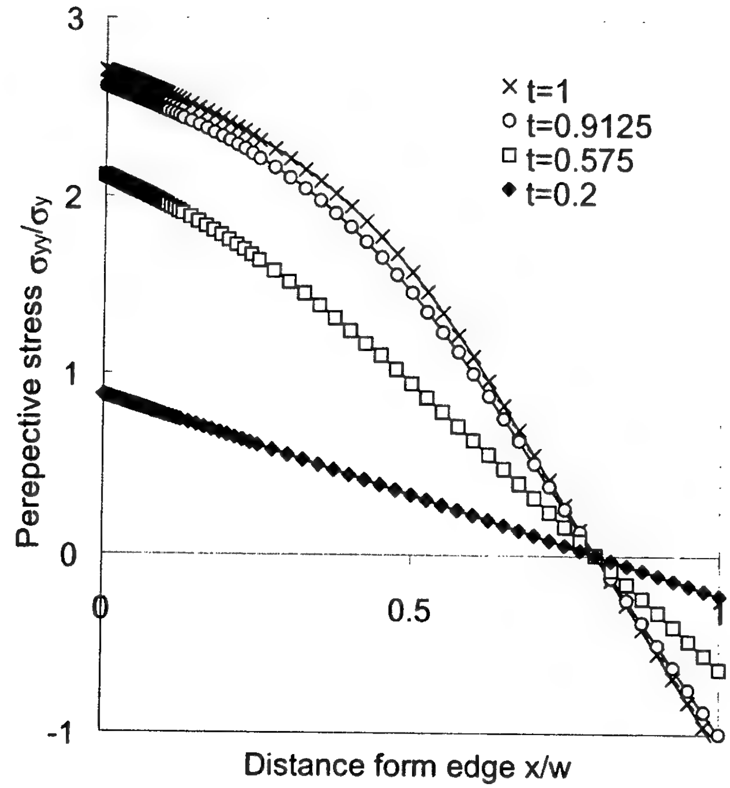


Figure 5 *A comparison of prospective stress between the predictions and FE results. Lines denote predictions, and symbols FE results.*

4. Strain-based Dugdale Models

The stress-based Dugdale model has been evaluated in (Wang, Chen and Rose, 2002a), and it was found that the standard implementation of this model is unable to adequately characterise the crack-tip opening displacement δ_M and the plastic zone size r_p. Consequently, a strain-based Dugdale approach was developed (Wang, Rose and Chen, 2002b) for a crack in a perfectly plastic material.

4.1 Stress-Based Dugdale Model

For a plate subjected to a combined bending and tension, with the prospective stress having been calculated, the crack-tip opening displacement can now be calculated using the following method.

According to Dugdale's cohesive zone concept the plastic zone size r_p of a crack length a is given by the root of the following equation,

$$K_{\sigma}(r_p) + K_{\sigma_0}(r_p) = 0 \tag{16}$$

If a represents the original crack size, and the r_p represents the plastic zone size in front of the crack tip, and then $c = a + r_p$. Eq (16) can be readily solved by iteration, given the stress distribution along the crack σ(x), here K_σ and K_{σ_0} are the stress-intensity factors for the two sub-problems, which can be written as:

$$K_\sigma(r_p) = \int_0^c \sigma(x) G(x,c) dx \tag{17}$$

and

$$K_{\sigma_0}(r_p) = -\alpha \int_a^c \sigma_0 G(x,c) dx \tag{18}$$

where, α is the plastic constraint factor: $\alpha = 1$ for plane stress and $\alpha = 1.73$ (Erwin, 1960) for plane strain. The function G denotes the stress-intensity factor of a crack of half-length c subjected to a pair of point forces acting at x. The Green function for an edge crack in a half-plane is (Tada, 1985),

$$G(x,c) = \frac{2}{\sqrt{\pi c}} \frac{1.3 - 0.3(x/c)^{5/4}}{[1-(x/c)^2]^{1/2}} \tag{19}$$

The total CTOD is the sum of the displacement Δ_σ due to the traction $\sigma_{yy}(x)$ and the displacement Δ_{σ_0} due to the cohesive stress σ_0:

$$\Delta = \Delta_\sigma + \Delta_{\sigma_0} \tag{19}$$

Where (Tada et al, 1985):

$$\Delta_\sigma = \frac{2}{E'} \int_a^c \int_0^b \sigma_{yy}(x) G(x,a) G(a,b) dx db \tag{20}$$

where b represents any point between the original crack tip a and current crack tip c, and

$$\Delta_{\sigma_0} = -\alpha \int_a^c \int_a^b \sigma_0 G(x,b) G(a,b) dx db \tag{22}$$

4.2 Strain-Based Dugdale Model

Like the Dugdale model, the BCS model (Bilby *et al*, 1963; Bilby and Eshelby, 1968) also uses a continuous distribution of dislocations to model both a crack and its associated plastic zone. Although the same mathematical formalism applies to both models, the physical interpretation of the BCS model suggests a plausible extension of the model for cracks under fully plastic loading. The dislocation density $D(x)$ in the BCS model is determined from the following singular integral equation,

$$\int_0^c D(t) Q(x,t) dt = \begin{cases} -\sigma_{yy}(x)/E' & (0 < x < a) \\ [-\sigma_{yy}(x) + \sigma_0)]/\mathrm{E}' & (a < x < c) \end{cases} \tag{23}$$

where the kernel $Q(x,t)$ has a Cauchy-type singularity for $x = t$, the model is restricted to cases where the applied stress $\sigma_{yy}(x)$ is within the elastic range. If we view the right hand side of Eq (23) in terms of the applied strain $\sigma_{yy}(x)/E'$, rather

than the applied stress, which controls the dislocation density, the model could still be applied to cases where the applied stress exceeds the yield stress, which would prevail in the absence of a crack. Similarly, the Dugdale model can be rewritten in terms of total strain in the same form as Eq (23) by defining hypothetical stress distributions σ^{*} and σ_0^{*} as follows,

$$\int_0^c D(t)Q(x,t)dt = \begin{cases} -\sigma^{*}(x)/E & (0<x<a) \\ [-\sigma^{*}(x)+\sigma_0^{*}(x)]/\mathrm{E} & (a<x<c) \end{cases} \tag{24a}$$

where

$$\sigma^{*}(x) = E\ \varepsilon_{ep}(x) \tag{24b}$$

$$\sigma_0^{*}(x) = \sigma_0 E\ \ /E_s(x) \tag{24c}$$

Equations (24a-c) are almost identical to those for the stress-based model provided that $\sigma^{*}(x)$ given by Eq (24b) is regarded as defining the applied stress which would prevail in the absence of a crack, and $\sigma_0^{*}(x)$ is interpreted as the new limit stress, which is now a function of position, through Eq (24c). Nevertheless, the extended-crack length c can still be determined iteratively from the condition,

$$K(c) = 0 \tag{25a}$$

with $\sigma_{yy}(x, y=0)$ now taken to be specified by

$$\sigma_{yy}(x, y=0) = \begin{cases} -\sigma^{*}(x) & (0 < x < a) \\ -\sigma^{*}(x) + \sigma_0^{*}(x) & (a < x < c) \end{cases} \tag{25b}$$

The advantage of this formulation is that convenient analytical approximations for the weight function $G(x,c)$ in Eq 24(a) are now available for a wide range of crack geometrises (Tada, 1985), even though exact analytical solutions are available for even fewer geometrise than for the dislocation kernel. For example, an exact analytical solution for $G(x,c)$ is not available for the case of a crack emanating from a circular hole, or even for the limiting case of an edge crack in a half-plane. However, for the latter case, Tada (1985) gives the convenient approximation of acceptable accuracy shown in Eq (19).

Given an explicit representation such as Eq (19) for $G(x,c)$, one can derive the extended-crack length c, and hence the plastic-zone size, $r_p = c - a$, for the strain-based model. The crack opening displacement $\delta(x)$ is related to the dislocation density through,

$$\delta_M(x) = u_y(x, y \to 0+) - u_y(x, y \to 0-), \tag{26a}$$

$$= \int_x^c D(t)dt \tag{26b}$$

which can be determined using the method outlined in Section 4.1.

4.3 Cohesive Stress

The application of the Dugdale model for plane strain requires the use of semi-empirical constraint factors that have been extensively investigated and discussed in previous work (Newman, 1982; 1998; Guo *et al*, 1999). A further discussion regarding the appropriate choice of constraint factor can be found in the work (Wang, Rose and Chen, 2002b).

The last remaining problem for the above strain-based Dugdale model of power law material is to determine the cohesive stress. Denoting σ_0 as the cohesive stress, one can simplify the stress-strain curve of the power law materials as elastic-perfectly plastic in which the yielding stress is assumed as the cohesive stress. This cohesive stress can be selected by using three methods: (1) Necking Stress Method: the cohesive stress equals to the necking stress; (2) Equal-Area Method: the cohesive stress is selected to give the same area below the stress-strain curve; and (3) HRR Method: the cohesive stress is selected according to a small-scale yielding (SSY) solution. These three methods are summarised in the following section.

Denoting σ_n as necking stress (Chakrabarty, 1987), which is the root of equation $\frac{d\sigma}{d\varepsilon} = 0$, the corresponding strain at which is $\varepsilon = 1/n$. Substituting this solution into the Eq (4), one can get:

$$\frac{1}{\varepsilon_Y n} = \frac{\sigma_n}{\sigma_Y} + \alpha\left(\frac{\sigma_n}{\sigma_Y}\right)^n \tag{27}$$

Solving the above equation, we can obtain the necking stress σ_n for a given strain. Simply we can directly use the necking stress as the cohesive stress, which is referred to as Necking Stress Method here. Rice [1968] proposed to use σ_n in conjunction with Dugdale model solution:

$$\delta_M = \frac{J}{\sigma_n} \tag{28}$$

If a solid curve denotes the original stress-strain curve of power law material shown in Fig (6), and a doted curve denotes a new stress-strain curve derived from the original stress-strain curve according to the Equal-Area Method, which means that the area below the solid curve should be equal to the area below the dotted line. As can be seen in Fig. (6), the new dotted curve represents an elastic-perfectly plastic curve in which the yielding stress is cohesive stress σ_0. Because the area under the solid curve equals to the area under the dotted curve, we get:

$$\frac{1}{2}E\sigma_0^{\ 2} + \sigma_0(\varepsilon_{\max} - \varepsilon_0) = \frac{1}{2}E\sigma_y^{\ 2} + \int_{\varepsilon_y}^{\varepsilon_{\max}} \sigma d\varepsilon \tag{29}$$

Here the maximum stress is assumed as the necking stress. Substituting the corresponding necking strain into the above equation:

$$\frac{1}{2}E(\sigma_0^{\ 2} - \sigma_y^{\ 2}) + \sigma_0(\frac{1}{n} - \varepsilon_0) = \int_{\varepsilon_y}^{1/n} \sigma d\varepsilon \tag{30}$$

From the above equation, the cohesive stress can be solved. This method is suitable for large scale yielding (LSY) condition, and we refer to it as the equal-area method.

Another method of obtaining the cohesive stress is to use the SSY solution. The crack-tip opening displacement CTOD is given by:

$$\delta_M = d_n \frac{J}{\sigma_Y} \tag{31}$$

and the relationship between J and K is given by

$$J = \frac{K^2}{E} \tag{32}$$

therefore, the final formulation of CTOD can be written by:

$$\delta_M = d_n \frac{K^2}{\sigma_Y E} \tag{33}$$

where σ_Y is the yielding stress, and d_n is the displacement variable which is a function of n and $(\frac{\sigma}{E})$. Recalling the elastic theorem, the formulation of CTOD is:

$$\delta_M = \frac{K^2}{E\sigma_Y} \tag{34}$$

Matching the form of Eq (34) of the elastic theorem, the plastic Eq (33) can also be rewritten as :

$$\delta_M = \frac{K^2}{\sigma_0 E} \tag{35}$$

Comparing Eq (33) and Eq (35), we can get the following formulation of cohesive stress:

$$\sigma_0 = \frac{\sigma_Y}{d_n} \tag{36}$$

The values of d_n can be found in Anderson (1985). This method is suitable for small scale yielding cases (SSY), and we call it HRR method. There are some differences between the results of LSY and SSY methods as shown in Fig. (7).

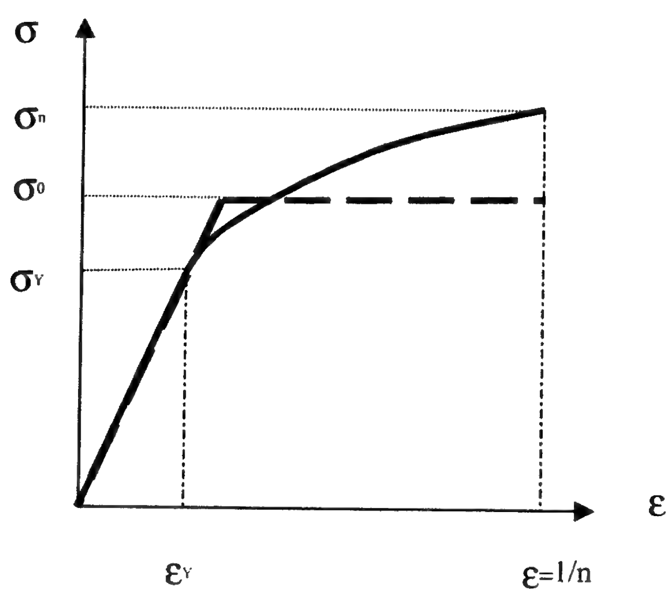


Figure 6 Definition of cohesive stress

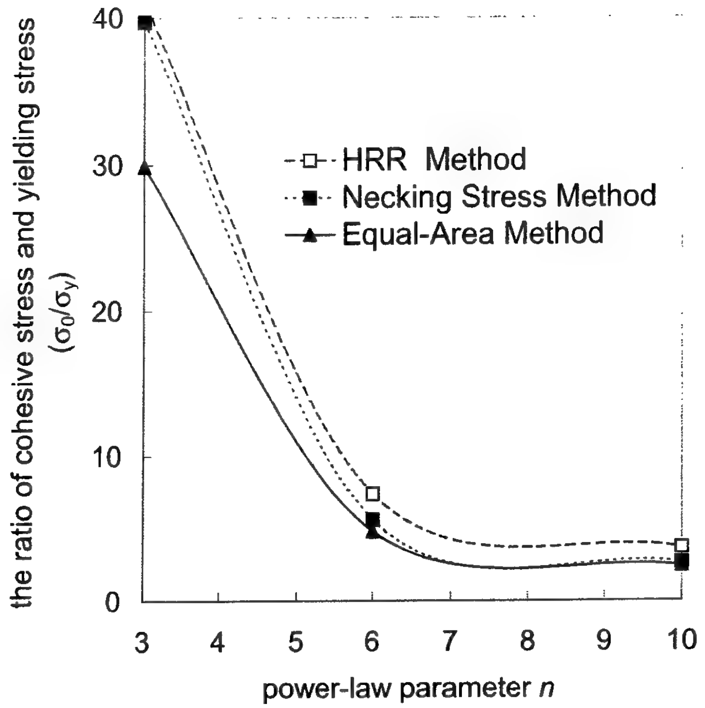


Figure 7 *Acomparison of cohesive stress by using the HRR Method, Necking Stress Method and Equal-Area Method.*

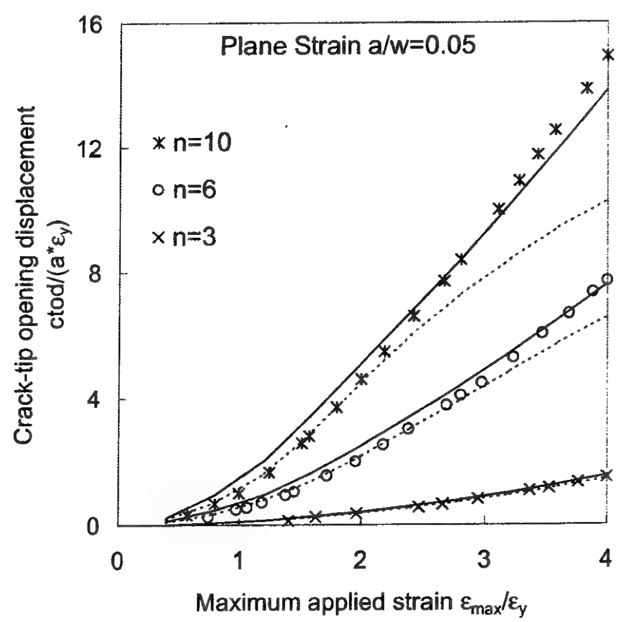


Figure 8 *Comparison between predictions of strain-based Dugdale model and finite element results for crack-tip opening displacement under plane strain conditions. The solid lines indicate predictions using the cohesive stresses obtained from the Equal-Area method and the dashed lines indicate predictions using cohesive stresses from the Necking Stress Method,and symbols indicate FE results.*

5. Results and Discussion

In this section, the predictive accuracy of the analytical models presented in Section 2 will be assessed against the computational (FE) results. Some issues will also be briefly discussed, along with recommendations on promising directions for future work.

The analytical estimates of crack-tip-opening displacement based on strain-based model compared with FE results are discussed here. The controlling parameter is the maximum applied strain $\varepsilon_{\max}$, normalised by the uniaxial yield strain $\varepsilon_Y = \sigma_Y / E$. In order to use the two-dimensional Dugdale model to characterise the complex three-dimensional plastic deformation of plane strain cracks, the limiting stress is chosen to represent a thickness-average of the normal stresses in the plastic zone. However, it has been reported that the Dugdale model with the stress-based constraint factor provides a reasonable estimate of the plastic zone size, but over-estimates the crack-tip opening displacement (Newman *et al*, 1995). Under large-scale yielding and gross-section yielding conditions, the significant under-estimation of the plastic zone size by the stress-based method indicates that, due to the loss of plastic constraint, a lower constraint factor would seem more appropriate (Wang 2000). Fig. 8 shows the results obtained from the strain-based Dugdale model of Section 3. The solid lines indicate predictions using the cohesive stresses obtained from the Equal-Area Method and the dashed lines indicate the predictions of the Necking Stress Method. It can be seen that former predictions are in better agreement with the FE results than the latter predictions. This confirms that the Equal-Area method of cohesive stress proposed in this work pertain to the problem of a small crack engulfed within a notch plastic zone for power-law strain hardening materials, rather than the usual choice of cohesive stress for small-scale yielding conditions at a crack tip.

Since the objective for the present work is to develop a practical approach for predicting fatigue crack growth rates for power-law materials at notch roots, when the crack is still small compared with the notch plastic zone, the significance of the strain-based Dugdale model in this context and the requirements for implementation of the present approach in practice will now be discussed, along with recommendations for promising directions for future work.

Two key parameters are required for the implementation of the strain-based Dugdale model, *viz,* (i) the limit stress σ_0, or equivalently the constraint factor α; and (ii) the appropriate secant modulus E_s. As indicated above, for arbitrary notch geometrise these would need to be determined from an FE analysis, but it would also be able to expect that adequate approximations could be developed for standard notch geometrise, as discussed by Wang *et al* (1999), thereby reducing the computational burden in routine applications. Another requirement is the availability of analytical expressions or approximations for the dislocation solution $Q(x;c)$ in Eq (9a), or equivalently for the weight function $G(x;c)$. As noted in Section 2, Tada's work (1985) has shown that analytical approximations for $G(x;c)$ are now available for a

wide range of notch and crack geometrises, so that practical implementation of the present approach would not be hindered by this requirement

This strain-based Dugdale model is still based on the superposition method which reduces the elastic-plastic problem into a superposition of two elastic solutions of a fictitious crack subjected to, respectively, a prospective stress that would exist in the absence of the crack, and a cohesive stress equal to the yield stress of the perfectly plastic material. In this work, the strain-based Dugdale model is extended to consider power-law materials by simplifying the constitutive equation of power law material into an effective elastic-perfectly plastic constitutive law, offering a simple yet efficient mean of characterising the plastic deformation at a crack tip. In particular, for power-law materials, the determination of cohesive stress has a significant impact on the results. As addressed in Section 4, there are three methods to determine the cohesive stress, (1) the necking stress method, (2) LSY method and (3) SSY method. In this work a good correlation has been found by using the cohesive stress determined by proposed LSY method, in which the maximum stress is selected as the necking stress, thus this approach is recommended to the large scale yielding cases. The bound of applicability of these different methods is not very clear. Therefore further work in this direction would be required.

6. Conclusion

A strain-based implementation of the Dugdale model has been extended for further applications involving a small crack engulfed within a notch plastic zone of power-law strain hardening materials. The predictive accuracy of this model has been evaluated by comparison with elasto-plastic finite element analysis. To retain in their simplest form of the key features of strain control and of a strain gradient that characterise the deformation at a notch root, this evaluation focused on the response of an edge crack subjected to a linearly varying strain distribution. The strain-based model was shown to give accurate values for δ_M for strain levels up to $\varepsilon_{\max}/\varepsilon_Y = 4$, with no evidence of significantly diminished accuracy with increasing strain, provided that the cohesive stress is determined by the proposed LSY method which is appropriate for the notch plastic deformation in the absence of a crack for power-law hardening materials.

7. References

ABAQUS (1998) User Manual, Version 5.8, Hibbitt, Karlsson and Sorensen, Inc, Rhode Island, US.

Anderson, TL Fracture Mechanics, 2nd Edition, CRC Press, Boca Raton, USA, 1995.

Al-Ani, A. M. and Hancock, J. W. (1991) J-Dominance of short crack in tension and bending, J. Mech. Phys. Solids, Vol. 39, 23-43.

Bilby, B. A., Cottrell, A. H. and Swinden, K. H. (1963) The spread of plastic yield from a notch, Proceedings of the Royal Society, London, 272, 304-314.

Bilby, B. A. and Eshelby, J. D. (1968) Dislocations and the theory of fracture, Fracture (edited by H. Liebowitz), Vol.1, Chapter 2, Academic Press, New York, 99-182.

Budiansky, B. and Hutchinson, J. W. (1978) Analysis of closure in fatigue crack growth, Journal of Applied Mechanics, Vol.45, 267-276.

Chakrabarty, J. (1987) Theory of Plasticity, McGraw-Hill Book Company, New York.

Cox, B.N. and Marshall, D. B. (1991) Stable and unstable solutions for bridged cracks in various specimens, Acta Metallurgica and Materialia, 39, 579-589.

Dugdale, D. S. (1960) Yielding of steel sheets containing slits, J. Mech. Phys. Solids, Vol.8, pp.100.

Edwards, P. R. and Newman, J. C. (1990) An AGARD supplemental test programme on the behaviour of short cracks under constant amplitude and aircraft spectrum loading, AGARD-R-767.

Foote, R. M. L., Mai, Y. W. and Cotterell, B. (1986) Crack growth resistance curves in strain softening materials, J. Mechanics and Physics of Solids, 34, 593-607.

Galatolo, R. and Lazzeri, L. (1996) Significance of short fatigue cracks, Aerotecnica Missilie Spazio, Vol.75, 34-46.

Guo, W., Wang, C. H. and Rose, L. R. F. (1999) On the influence of cross sectional thickness on fatigue crack growth, Fatigue and Fracture of Engineering Materials and Structures, Vol.22, 437-444.

Gowda, C.V.B., Topper, T.H. and Leis, B. N. (1972) Crack initiation and propagation in notched plates subject to cyclic inelastic strain, International Conference Mechanical Behaviour of Materials, Kyoto, Japan, 11, pp.187-198.

Haddaa, Et. Smith, M.H. and Topper, T.H. (1978) A strain-based intensity factor solution for short fatigue cracks initiating from notches, ASTM STP 677, pp274-89.

Newman, J.C. Jr. (1982) A non-linear fracture mechanics approach to the growth of small cracks, AGARD-CP-328, paper 6.

Newman, J.C. Jr. (1992) FASTRANII- A fatigue crack growth structure analysis program, NASA Technical Memorandum, 104159.

Newman, J. C. Jr. (1998) The merging of fatigue and fracture mechanics concepts: a historical perspective, Progress in Aerospace Sciences, Vol.34, 347-390.

Rich, T and Roberts (1968) Plastic enclave size for internal cracks emanating from circular cavities within elastic plates, Engineering Fracture Mechanics, Vol.1, 167-173.

Rice J. R. (1968) Mathematical analysis in the mechanics of fracture, Fracture, Vol. II , US, 191-311.

Rose, L. R. F. and Wang, C. H. (2000) Self-similar analysis of plasticity-induced closure of small fatigue cracks, J. Mech. Phys. Solids. Vol. 49, 401-429.

Saff, C. R. (1984) Crack growth retardation and acceleration models, *in* Damage Tolerance of Metallic Structures: Analysis Methods and Application, ASTM STP 842, J. B. Chang and J. L. Rudd, Eds., American Society for Testing and Materials, 36-49.

Shih, C. F. (1981) Relationships between the J–integral and crack opening displacement for stationary and extending cracks, J. Mech. Phys. Solids. Vol. 29, 305-326.

Shin, C. S. (1994) Fatigue crack growth from stress concentrations and fatigue life prediction in notched components, Handbook of Fatigue Crack Propagation in Metallic Structures, Andrea Carpinteri (editor), Elsevier Science B. V., UK.

Smith, R. A. and Miller, K. J. (1978) Prediction of fatigue regimes in notched components, Int. J. Mech. Science, Vol.20, 201-206.

Suresh, S. (1991) Fatigue of Materials, Cambridge Solid State Science Series, Cambridge University Press, UK.

Tada, H., Paris, P. C. And Irwin, G. R. (1985) The Stress Analysis of Cracks Handbook, Del Research Corp., Hellertown PA, USA.

Tanaka, K., Hoshide, T. and Sakai, N. (1984) Mechanics of fatigue crack propagation by crack-tip plastic blunting, Engineering Fracture Mechancis, Vol.19, 805-825.

Wang, C. H, Chen, G. X. and Rose, L. R. F. (2002a) A critical evaluation of superposition methods for cracks in grossly plastic gradient fields, J. of Engineering Fracture Mechanics, Vol. 69, 633-646.

Wang, C. H, Rose, L. R. F. and Chen, G. X. (2002b) A Strain-based Dugdale Model for cracks under generally yielding condition, International Journal of Fracture, Vol. 113, 77-104

Wang, C. H, Chen, G. X. and Rose, L. R. F. (1999) An assessment of superposition methods under generally yielding conditions, Proceedings of International Workshop on Fracture Mechanics and Advanced Engineering Materials, University of Sydney, Australia, pp.192-199.

Wang, C. H., Guo, W. and Rose, L. R. F. (1999) A method for determining the elastic-plastic response ahead of a notch tip, Journal of Engineering Materials and Technology, Vol.121, 313-320.

Wang, C. H. and Rose, L. R. F. (1999) Crack-tip plastic blunting under gross-section yielding and implications for short crack growth, Fatigue and Fracture of Engineering Materials and Structures, Vol.22, 761-773.

Wang, G. S. and Blom, A. F. (1991) A strip model for fatigue crack growth predictions under general load conditions, Engineering Fracture Mechanics, Vol.40, 507-533.

DISTRIBUTION LIST

A Strain-based Cohesive Zone Model for a Crack in a Power Law Material under Grossly Plastic Conditions

G. X. Chen, C. H. Wang, and L. R. F. Rose

AUSTRALIA

DEFENCE ORGANISATION

S&T Program

Chief Defence Scientist
FAS Science Policy
AS Science Corporate Management
Director General Science Policy Development
shared copy
Counsellor Defence Science, London (Doc Data Sheet)
Counsellor Defence Science, Washington (Doc Data Sheet)
Scientific Adviser to MRDC Thailand (Doc Data Sheet)
Scientific Adviser Joint
Navy Scientific Adviser (Doc Data Sheet and distribution list only)
Scientific Adviser - Army (Doc Data Sheet and distribution list only)
Air Force Scientific Adviser
Director Trials

Aeronautical and Maritime Research Laboratory
Director

Chief of Airframes and Engines
Research Leader Aircraft Composite Structure
Research Leader Aircraft Structural Integrity
M. Heller
W. Hu
Author(s): G. X. Chen (5 copies)
C. H. Wang (3 copies)
RLFM (3 copies)

DSTO Library and Archives

Library Fishermans Bend
Library Edinburgh (1 copy)
Australian Archives
US Defense Technical Information Center, 2 copies
UK Defence Research Information Centre, 2 copies
Canada Defence Scientific Information Service, 1 copy
NZ Defence Information Centre, 1 copy
National Library of Australia, 1 copy

Capability Systems Staff

Director General Maritime Development (Doc Data Sheet only)
Director General Aerospace Development (Doc Data Sheet only)

Knowledge Staff

Director General Command, Control, Communications and Computers (DGC4) (Doc Data Sheet only)

Army

ABCA National Standardisation Officer Land Warfare DevelopmentSector, Tobruk Barracks,PUCKAPUNYAL VIC 3662, (4 copies)

SO (Science), Deployable Joint Force Headquarters (DJFHQ) (L), MILPO Gallipoli Barracks, Enoggera QLD 4052 (Doc Data Sheet only)

Intelligence Program

DGSTA Defence Intelligence Organisation

Manager, Information Centre, Defence Intelligence Organisation

Corporate Support Program

Library Manager, DLS-Canberra

Ms Sam Doran, Defence Library Service - Sydney West

UNIVERSITIES AND COLLEGES

Australian Defence Force Academy

Library

Head of Aerospace and Mechanical Engineering

Serials Section (M list), Deakin University Library, Geelong, 3217

Hargrave Library, Monash University (Doc Data Sheet only)

Librarian, Flinders University

OTHER ORGANISATIONS

NASA (Canberra)

AusInfo

OUTSIDE AUSTRALIA

ABSTRACTING AND INFORMATION ORGANISATIONS

Library, Chemical Abstracts Reference Service

Engineering Societies Library, US

Materials Information, Cambridge Scientific Abstracts, US

Documents Librarian, The Center for Research Libraries, US

INFORMATION EXCHANGE AGREEMENT PARTNERS

Acquisitions Unit, Science Reference and Information Service, UK

Library - Exchange Desk, National Institute of Standards and Technology, US

National Aerospace Laboratory, Japan

National Aerospace Laboratory, Netherlands

SPARES (5 copies)

Total number of copies: **58**

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION DOCUMENT CONTROL DATA	1. PRIVACY MARKING/CAVEAT (OF DOCUMENT)

2. TITLE	3. SECURITY CLASSIFICATION (FOR UNCLASSIFIED REPORTS THAT ARE LIMITED RELEASE USE (L) NEXT TO DOCUMENT CLASSIFICATION)
A Strain-based Cohesive Zone Model for a Crack in a Power-Law Material under Grossly Plastic Conditions	Document (U) Title (U) Abstract (U)
4. AUTHOR(S) G. X. Chen, C. H. Wang, and L. R. F. Rose	5. CORPORATE AUTHOR Aeronautical and Maritime Research Laboratory 506 Lorimer St Fishermans Bend Victoria 3207 Australia

6a. DSTO NUMBER	6b. AR NUMBER	6c. TYPE OF REPORT	7. DOCUMENT DATE
DSTO-RR-0235	AR-012-297	Research Report	April, 2002

8. FILE NUMBER	9. TASK NUMBER	10. TASK SPONSOR	11. NO. OF PAGES	12. NO. OF REFERENCES
M1/9/869	LRR 01/159	LRR	18	34

13. URL on the World Wide	14. RELEASE AUTHORITY
http://www.dsto.defence.gov.au/corporate/reports/DSTO-RR-0235.pdf	Chief, Airframes and Engines Division

15. SECONDARY RELEASE STATEMENT OF THIS DOCUMENT

Approved for public release

OVERSEAS ENQUIRIES OUTSIDE STATED LIMITATIONS SHOULD BE REFERRED THROUGH DOCUMENT EXCHANGE, PO BOX 1500, EDINBURGH, SA 5111

16. DELIBERATE ANNOUNCEMENT

No Limitations

17. CITATION IN OTHER DOCUMENTS Yes

18. DEFTEST DESCRIPTORS

crack, stress analysis, plasticity

19. ABSTRACT

To develop an analytical method for quantifying the growth behaviour of short cracks embedded in notch plastic zones for power law strain hardening materials, a strain-based cohesive zone model is proposed in which the conventional equilibrium equation in the stress-based model is replaced by strain compatibility. A comparison with finite element results shows that this strain-based model provides accurate values of the crack-tip-opening displacement for applied strains up to four times the yield strain under general yielding conditions. Furthermore, it is shown that the cohesive stress determined by a method proposed in this work gives better results than the existing method, which are appropriate only for small-scale yielding conditions.

RESEARCH REPORT DSTO-RR-0235 AR-012-297 APRIL 2002

DEFENCE
SCIENCE & TECHNOLOGY

AERONAUTICAL AND MARITIME RESEARCH LABORATORY
506 LORIMER STREET FISHERMANS BEND VICTORIA 3207 AUSTRALIA
TELEPHONE (03) 9626 7000